## Chapter 6



## Costs

## Basic Concepts of Costs

- Opportunity cost is the cost of a good or service as measured by the alternative uses that are foregone by producing the good or service.
- If 15 bicycles could be produced with the materials used to produce an automobile, the opportunity cost of the automobile is 15 bicycles.
- The price of a good or service often may reflect its opportunity cost.


## Basic Concepts of Costs

- Accounting cost is the concept that goods or services cost what was paid for them.
- Economic cost is the amount required to keep a resource in its present use; the amount that it would be worth in its next best alternative use.


## Labor Costs

- Like accountants, economists regard the payments to labor as an explicit cost.
- Labor services (worker-hours) are purchased at an hourly wage rate (w): The cost of hiring one worker for one hour.
- The wage rate is assumed to be the amount workers would receive in their next best alternative employment.


## Capital Costs

- While accountants usually calculate capital costs by applying some depreciation rule to the historical price of the machine, economists view this amount as a sunk cost.
- A sunk cost is an expenditure that once made cannot be recovered.
- These costs do not focus on foregone opportunities.


## Capital Costs

- Economists consider the cost of a machine to be the amount someone else would be willing to pay for its use.
- The cost of capital services (machine-hours) is the rental rate ( $\mathbf{v}$ ) which is the cost of hiring one machine for one hour.
- This is an implicit cost if the machine is owned by the firm.


## Two Simplifying Assumptions

- The firm uses only two inputs: labor (L, measured in labor hours) and capital (K, measured in machine hours).
- Entrepreneurial services are assumed to be included in the capital costs.
- Firms buy inputs in perfectly competitive markets so the firm faces horizontal supply curves at prevailing factor prices.


## Economic Profits and Cost Minimization

- Total costs = TC = wL + vK.
- Assuming the firm produces only one output, total revenue equals the price of the product $(P)$ times its total output $[q=f(K, L)$ where $f(K, L)$ is the firm's production function].


## Economic Profits and Cost Minimization

- Economic profits $(\pi)$ is the difference between a firm's total revenues and its total economic costs.

$$
\begin{aligned}
\pi & =\text { Total revenues }- \text { Total costs } \\
& =P q-w L-v K \\
& =P f(K, L)-w L-v K .
\end{aligned}
$$

## Cost-Minimizing Input Choice

- Assume, for purposes of this chapter, that the firm has decided to produce a particular output level (say, $q_{1}$ ).
- The firm's total revenues are $P \cdot q_{1}$.
- How the firm might choose to produce this level of output at minimal costs is the subject of this chapter.


## Cost-Minimizing Input Choice

- Cost minimization requires that the marginal rate of technical substitution (RTS) of L for K equals the ratio of the inputs' costs, w/v:

$$
R T S(\text { of } \mathrm{L} \text { for } \mathrm{K})=\frac{w}{v}
$$

## Graphic Presentation

- The isoquant $\mathrm{q}_{1}$ shows all combinations of K and $L$ that are required to produce $q_{1}$.
- The slope of total costs, $\mathrm{TC}=\mathrm{wL}+\mathrm{vK}$, is w/v.
- Lines of equal cost will have the same slope so they will be parallel.
- Three equal total costs lines, labeled $\mathrm{TC}_{1}$, $\mathrm{TC}_{2}$, and $\mathrm{TC}_{3}$ are shown in Figure 6.1.



## Graphic Presentation

- The minimum total cost of producing $\mathrm{q}_{1}$ is $\mathrm{TC}_{1}$ (since it is closest to the origin).
- The cost-minimizing input combination is $\mathrm{L}^{*}, \mathrm{~K}^{*}$ which occurs where the total cost curve is tangent to the isoquant.
- At the point of tangency, the rate at which the firm can technically substitute L for K (the RTS) equals the market rate ( $\mathrm{w} / \mathrm{v}$ ).


## An Alternative Interpretation

- From Chapter $5 R T S($ of L for K$)=\frac{M P_{L}}{M P_{K}}$.
- Cost minimization requires
- or, rearranging

$$
R T S=\frac{M P_{L}}{M P_{K}}=\frac{w}{v},
$$

$$
\frac{M P_{L}}{w}=\frac{M P_{K}}{v}
$$

## The Firm's Expansion Path

- A similar analysis could be performed for any output level (q).
- If input costs ( $w$ and $v$ ) remain constant, various cost-minimizing choices can be traces out as shown in Figure 6.2.
- For example, output level $\mathrm{q}_{1}$ is produced using $\mathrm{K}_{1}$, $\mathrm{L}_{1}$, and other cost-minimizing points are shown by the tangency between the total cost lines and the isoquants.


## The Firm's Expansion Path

- The firm's expansion path is the set of costminimizing input combinations a firm will choose to produce various levels of output (when the prices of inputs are held constant).
- Although in Figure 6.2, the expansion path is a straight line, that is not necessarily the case.


## FIGURE 6.2: Firm's Expansion Path



## Cost Curves

- A firm's expansion path shows how minimumcost input use increases when the level of output expands.
- With this it is possible to develop the relationship between output levels and total input costs.
- These cost curves are fundamental to the theory of supply.


## Cost Curves

- Figure 6.3 shows four possible shapes for cost curves.
- In Panel a, output and required input use is proportional which means doubling of output requires doubling of inputs. This is the case when the production function exhibits constant returns to scale.


## FIGURE 6.3: Possible Shapes of the Total Cost Curve


(a) Constant Returns to Scale

## Cost Curves

- Panels $b$ and $c$ reflect the cases of decreasing and increasing returns to scale, respectively.
- With decreasing returns to scale the cost curve is convex, while the it is concave with increasing returns to scale.
- Decreasing returns to scale indicate considerable cost advantages from large scale operations.


## FIGURE 6.3: Possible Shapes of the Total Cost Curve



## Cost Curves

- Panel d reflects the case where there are increasing returns to scale followed by decreasing returns to scale.
- This might arise because internal co-ordination and control by managers is initially underutilized, but becomes more difficult at high levels of output.
- This suggests an optimal scale of output.


## FIGURE 6.3: Possible Shapes of the Total Cost Curve



## Average Costs

$$
\text { Average cost }=A C=\frac{T C}{q}
$$

- Average cost is total cost divided by output; a common measure of cost per unit.
- If the total cost of producing 25 units is $\$ 100$, the average cost would be

$$
A C=\frac{\$ 100}{25}=\$ 4
$$

## Marginal Cost

## Marginal cost $=M C=\frac{\text { Change in TC }}{\text { Change in } \mathrm{q}}$

- The additional cost of producing one more unit of output is marginal cost.
- If the cost of producing 24 units is $\$ 98$ and the cost of producing 25 units is $\$ 100$, the marginal cost of the $25^{\text {th }}$ unit is $\$ 2$.


## Marginal Cost Curves

- Marginal costs are reflected by the slope of the total cost curve.
- The constant returns to scale total cost curve shown in Panel a of Figure 6.3 has a constant slope, so the marginal cost is constant as shown by the horizontal marginal cost curve in Panel a of Figure 6.4.


## FIGURE 6.4: Average and Marginal Cost Curves

AC, MC \(\begin{gathered}AC, MC<br>0 \quad \begin{array}{c}Quantity<br>per week\end{array}<br>(a) Constant Returns to Scale\end{gathered}\)

## Marginal Cost Curves

- With decreasing returns to scale, the total cost curve is convex (Panel b of Figure 6.3).
- This means that marginal costs are increasing which is shown by the positively sloped marginal cost curve in Panel b of Figure 6.4.


## FIGURE 6.4: Average and Marginal Cost Curves



## Marginal Cost Curves

- Increasing returns to scale results in a concave total cost curve (Panel c of Figure 6.3).
- This causes the marginal costs to decrease as output increases as shown in the negatively sloped marginal cost curve in Panel c of Figure 6.4.


## FIGURE 6.4: Average and Marginal Cost Curves



AC, MC

(c) Increasing Returns to Scale

## Marginal Cost Curves

- When the total cost curve is first concave followed by convex as shown in Panel d of Figure 6.3, marginal costs initially decrease but eventually increase.
- Thus, the marginal cost curve is first negatively sloped followed by a positively sloped curve as shown in Panel d of Figure 6.4.


## FIGURE 6.4: Average and Marginal Cost Curves



## Average Cost Curves

- If a firm produces only one unit of output, marginal cost would be the same as average cost
- Thus, the graph of the average cost curve begins at the point where the marginal cost curve intersects the vertical axis.


## Average Cost Curves

- For the constant returns to scale case, marginal cost never varies from its initial level, so average cost must stay the same as well.
- Thus, the average cost curve are the same horizontal line as shown in Panel a of Figure 6.4.


## Average Cost Curves

- With convex total costs and increasing marginal costs, average costs also rise as shown in Panel b of Figure 6.4.
- Because the first few units are produced at low marginal costs, average costs will always $b$ less than marginal cost, so the average cost curve lies below the marginal cost curve.


## Average Cost Curves

- With concave total cost and decreasing marginal costs, average costs will also decrease as shown in Panel c in Figure 6.4.
- Because the first few units are produced at relatively high marginal costs, average is less than marginal cost, so the average cost curve lies below the marginal cost curve.


## Average Cost Curves

- The U-shaped marginal cost curve shown in Panel d of Figure 6.4 reflects decreasing marginal costs at low levels of output and increasing marginal costs at high levels of output.
- As long as marginal cost is below average cost, the marginal will pull down the average.


## Average Cost Curves

- When marginal costs are above average cost, the marginal pulls up the average.
- Thus, the average cost curve must intersect the marginal cost curve at the minimum average cost; $q^{*}$ in Panel d of Figure 6.4.
- Since $q^{*}$ represents the lowest average cost, it represents an "optimal scale" of production for the firm.


## Distinction between the Short Run and the Long Run

- The short run is the period of time in which a firm must consider some inputs to be absolutely fixed in making its decisions.
- The long run is the period of time in which a firm may consider all of its inputs to be variable in making its decisions.


## Holding Capital Input Constant

- For the following, the capital input is assumed to be held constant at a level of $\mathrm{K}_{1}$, so that, with only two inputs, labor is the only input the firm can vary.
- As examined in Chapter 5, this implies diminishing marginal productivity to labor.


## Types of Short-Run Costs

- Fixed costs; costs associated with inputs that are fixed in the short run.
- Variable costs; costs associated with inputs that can be varied in the short run.


## Input Inflexibility and Cost Minimization

- Since capital is fixed, short-run costs are not the minimal costs of producing variable output levels.
- Assume the firm has fixed capital of $\mathrm{K}_{1}$ as shown in Figure 6.5.
- To produce $q_{0}$ of output, the firm must use $L_{1}$ units of labor, with similar situations for $q_{1}, L_{1}$, and $\mathrm{q}_{2}, \mathrm{~L}_{2}$.


## FIGURE 6.5: "Nonoptimal" Input Choices Must Be Made in the Short Run



## Input Inflexibility and Cost Minimization

- The cost of output produced is minimized where the RTS equals the ratio of prices, which only occurs at $\mathrm{q}_{1}, \mathrm{~L}_{1}$.
- $Q_{0}$ could be produce at less cost if less capital than $\mathrm{K}_{1}$ and more labor than $\mathrm{L}_{0}$ were used.
- $Q_{2}$ could be produced at less cost if more capital than $\mathrm{K}_{1}$ and less labor than $\mathrm{L}_{2}$ were used.


## Per-Unit Short-Run Cost Curves

Short - run average cost $=S A C=\frac{S T C}{q}$ and

Short - run marginal cost $=S M C=\underline{\text { Change in STC }}$ Change in q

## Per-Unit Short-Run Cost Curves

- Having capital fixed in the short run yields a total cost curve that has both concave and convex sections, the resulting short-run average and marginal cost relationships will also be U-shaped.
- When SMC < SAC, average cost is falling, but when SMC > SAC average cost increase.


## Relationship between Short-Run and Long-Run per-Unit Cost Curves

- Figure 6.6 shows all cost relationships for a firm that has U-shaped long-run average and marginal cost curves.
- At output level q* long-run average costs are minimized and MC = AC.
- Associated with $\mathrm{q}^{*}$ is a certain level of capital, $K^{*}$.


## FIGURE 6.6: Short-Run and Long-Run Average and Marginal Cost Curves at Optimal Output Level



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## Relationship between Short-Run and Long-Run per-Unit Cost Curves

- In the short-run, when the firm using $\mathrm{K}^{*}$ units of capital produces $q^{*}$, short-run and long-run total costs are equal.
- In addition, as shown in Figure 6.6

$$
A C=M C=S A C\left(K^{*}\right)=S M C\left(K^{*}\right)
$$

- For output above q* short-run costs are higher than long-run costs. The higher per-unit costs reflect the facts that K is fixed.


## Shifts in Cost Curves

- Any change in economic conditions that affects the expansion path will also affect the shape and position of the firm's cost curves.
- Three sources of such change are:
- change in input prices
- technological innovations, and
- economies of scope.


## Changes in Input Prices

- A change in the price of an input will tilt the firm's total cost lines and alter its expansion path.
- For example, a rise in wage rates will cause firms to use more capital (to the extent allowed by the technology) and the entire expansion path will rotate toward the capital axis.


## Changes in Input Prices

- Generally, all cost curves will shift upward with the extent of the shift depending upon how important labor is in production and how successful the firm is in substituting other inputs for labor.
- With important labor and poor substitution possibilities, a significant increase in costs will result.


## Technological Innovation

- Because technological advances alter a firm's production function, isoquant maps as well as the firm's expansion path will shift when technology changes.
- Unbiased improvements would shift isoquants toward the origin enabeling firms to produce the same level of output with less of all inputs.


## Technological Innovation

- Technological change that is biased toward the use of one input will alter isoquant maps, shift expansion paths, and affect the shape and location of cost curves.
- For example, if workers became more skilled, this would save only on labor input.


## Economies of Scope

- Economies of scope is the reduction in the costs of one product of a multiproduct firm when the output of another product is increased.
- For example, when hospitals do many surgeries of one type, it may have cost advantages in doing other types because of the similarities in equipment and operating personnel used.


## A Numerical Example

- Assume Hamburger Heaven (HH) can hire workers at $\$ 5$ per hour and it rents all of its grills for $\$ 5$ per hour.
- Total costs for HH during one hour are

$$
T C=5 K+5 L
$$

where $K$ and $L$ are the number of grills and the number of workers hired during that hour, respectively.

## A Numerical Example

- Suppose HH wishes to produce 40 hamburgers per hour.
- Table 6.1 repeats the various ways HH can produce 40 hamburgers per hour.
- This shows that total costs are minimized when $\mathrm{K}=\mathrm{L}=4$.
- Figure 6.10 shows the cost-minimizing tangency.


## TABLE 6.1: Total Costs of Producing 40 Hamburgers per Hour

| Output $(\boldsymbol{q})$ | Workers $(\boldsymbol{L})$ | Grills $(\mathbf{K})$ | Total Cost (TC) |
| :---: | :---: | :---: | :---: |
| 40 | 1 | 16.0 | $\$ 85.00$ |
| 40 | 2 | 8.0 | 50.00 |
| 40 | 3 | 5.3 | 41.50 |
| 40 | 4 | 4.0 | 40.00 |
| 40 | 5 | 3.2 | 41.00 |
| 40 | 6 | 2.7 | 43.50 |
| 40 | 7 | 2.3 | 46.50 |
| 40 | 8 | 2.0 | 50.00 |
| 40 | 9 | 1.8 | 54.00 |
| 40 | 10 | 1.6 | 58.00 |

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## Figure 6.7: Cost-Minimizing Input Choice for 40 Hamburgers per Hour



## Long-Run cost Curves

- HH's production function is constant returns to scale so
- As long as $w=v=\$ 5$, all of the cost minimizing tangencies will resemble the one shown in Figure 6.10 and long-run cost minimization will require $K=L$.


## Long-Run cost Curves

- This situation, resulting from constant returns to scale, is shown in Figure 6.8.
- HH's long-run total cost function is a straight line through the origin as shown in Panel a.
- Its long-run average and marginal costs are constant at $\$ 1$ per burger as shown in Panel b.


## FIGURE 6.8: Total, Average, and Marginal Cost Curves


(a) Total Costs
(b) Average and Marginal Costs

## Short-Run Costs

- Table 6.2 repeats the labor input required to produce various output levels holding grills fixed at 4.
- Diminishing marginal productivity of labor causes costs to rise rapidly as output expands.
- Figure 6.9 shows the short-run average and marginal costs curves.


## TABLE 6.2: Short-Run Costs of Hamburger Production

| Output $(\boldsymbol{q})$ | Workers <br> $(\boldsymbol{L})$ | Grills <br> $(\boldsymbol{K})$ | Total Cost <br> $($ STC $)$ | Average <br> Cost $($ SAC $)$ | Marginal <br> Cost $($ SMC $)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 0.25 | 4 | $\$ 21.25$ | $\$ 2.125$ | - |
| 20 | 1.00 | 4 | 25.00 | 1.250 | $\$ 0.50$ |
| 30 | 2.25 | 4 | 31.25 | 1.040 | 0.75 |
| 40 | 4.00 | 4 | 40.00 | 1.000 | 1.00 |
| 50 | 6.25 | 4 | 51.25 | 1.025 | 1.25 |
| 60 | 9.00 | 4 | 65.00 | 1.085 | 1.50 |
| 70 | 12.25 | 4 | 81.25 | 1.160 | 1.75 |
| 80 | 16.00 | 4 | 100.00 | 1.250 | 2.00 |
| 90 | 20.25 | 4 | 121.25 | 1.345 | 2.25 |
| 100 | 25.00 | 4 | 145.00 | 1.450 | 2.50 |

## FIGURE 6.12: Short-Run and Long-Run Average and Marginal Cost Curves for Hamburger Heaven



