Appendix to Chapter 1

Mathematics Used in Microeconomics

Functions of One Variable

- **Variables**: The basic elements of algebra, usually called X, Y, and so on, that may be given any numerical value in an equation
- **Functional notation**: A way of denoting the fact that the value taken on by one variable (Y) depends on the value taken on by some other variable (X) or set of variables

$$Y = f(X)$$

This is read "*Y is a function of X*"

Independent and Dependent Variables

- **Independent Variable**: In an algebraic equation, a variable that is unaffected by the action of another variable and may be assigned any value
- **Dependent Variable**: In algebra, a variable whose value is determined by another variable or set of variables

Two Possible Forms of Functional Relationships

- Y is a ***linear function*** of X

$$Y = a + bX$$

  - Table 1.A.1 shows some value of the linear function Y = 3 + 2X
- Y is a ***nonlinear function*** of X
  - This includes X raised to powers other than 1
  - Table 1.A.1 shows some values of a quadratic function $Y = -X^2 + 15X$

Table 1A.1: Values of X and Y for Linear and Quadratic Functions

Linear Function		Quadratic Function	
x	Y = *f(X)* = 3 + 2X	x	Y = *f(X)* = $-X^2 + 15X$
-3	-3	-3	-54
-2	-1	-2	-34
-1	1	-1	-16
0	3	0	0
1	5	1	14
2	7	2	26
3	9	3	36
4	11	4	44
5	13	5	50
6	15	6	54

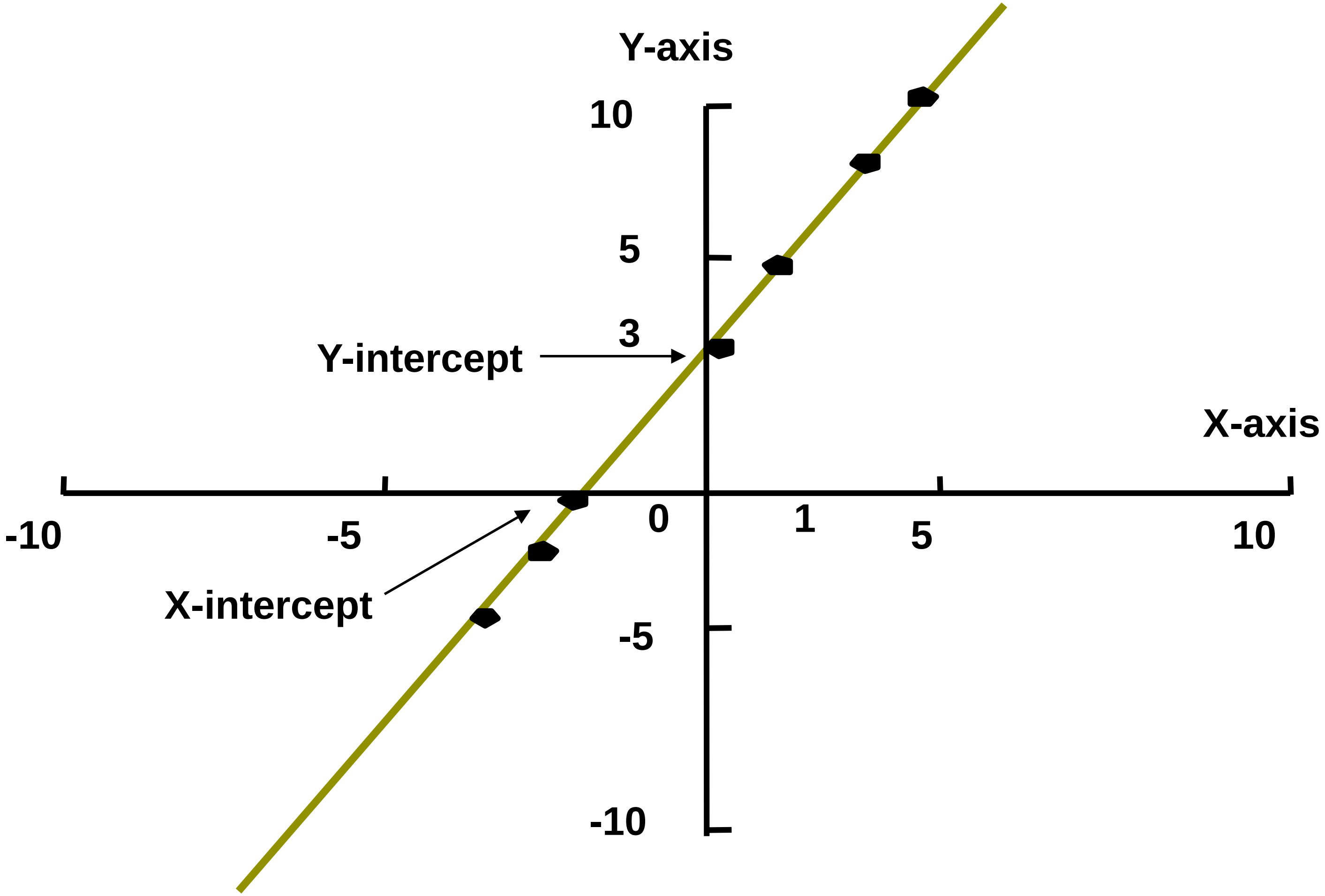
Graphing Functions of One Variable

- Graphs are used to show the relationship between two variables
- Usually the dependent variable (Y) is shown on the vertical axis and the independent variable (X) is shown on the horizontal axis
  - However, on supply and demand curves, this approach is reversed

Linear Function

- A linear function is an equation that is represented by a straight-line graph
- Figure 1A.1 represents the linear function $Y=3+2X$
- As shown in Figure 1A.1, linear functions may take on both positive and negative values

Figure 1A.1: Graph of the Linear Function Y = 3 + 2X



Intercept

- The general form of a linear equation is

$$Y = a + bX$$

- The Y-**intercept** is the value of Y when when X equals 0
  – Using the general form, when X = 0, Y = a, so this is the intercept of the equation

Slopes

- The **slope** of any straight line is the ratio of the change in Y (the dependent variable) to the change in X (the independent variable)
- The slope can be defined mathematically as

$$\text{Slope} = \frac{\text{Change in Y}}{\text{Change in X}} = \frac{\Delta Y}{\Delta X}$$

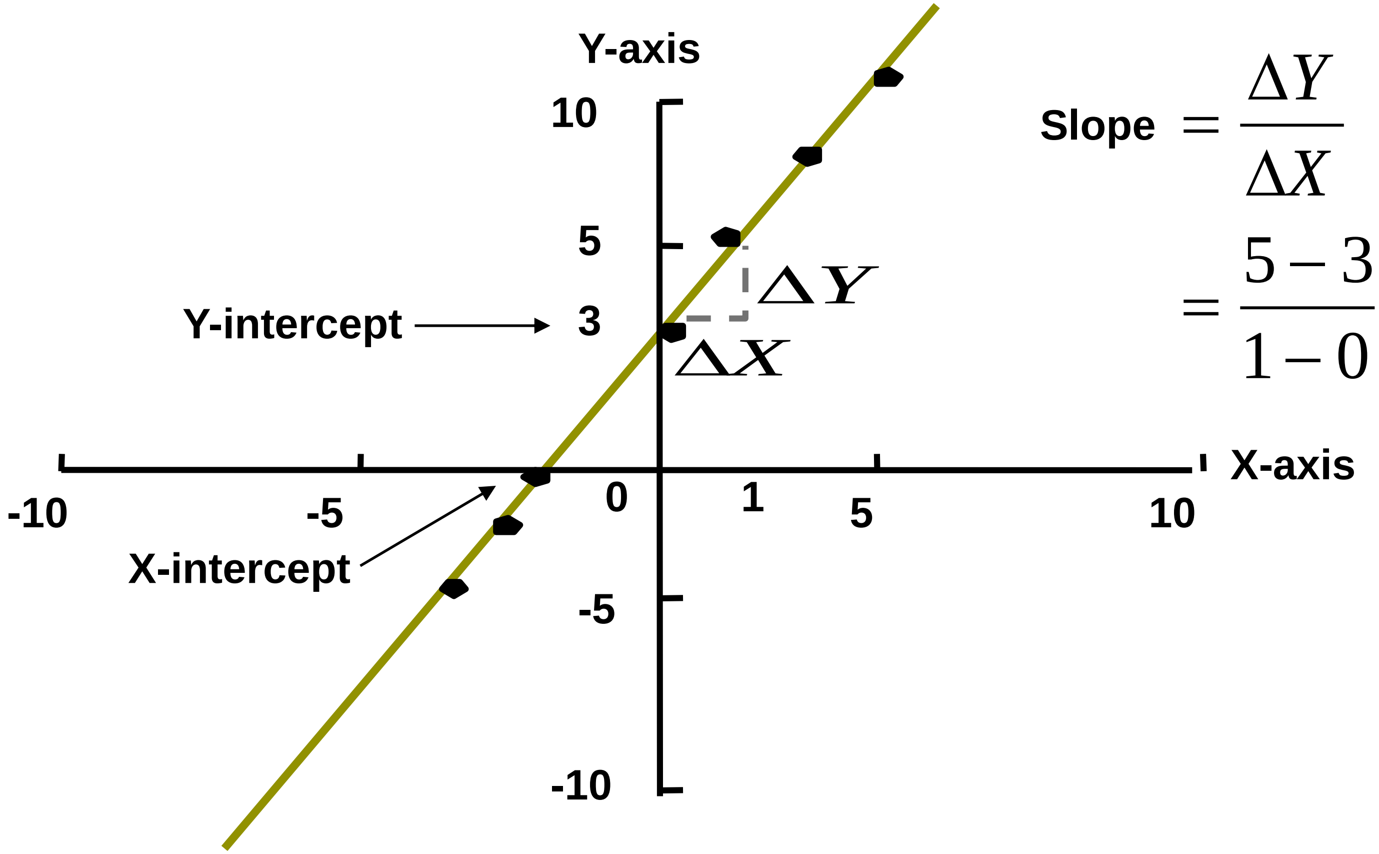
- where Δ means “change in”
- It is the direction of a line on a graph.

Slopes

- For the equation Y = 3 + 2X the slope equals 2 as can be seen in Figure 1A.1 by the dashed lines representing the changes in X and Y
- As X increases from 0 to 1, Y increases from 3 to 5

$$\text{Slope} = \frac{\Delta Y}{\Delta X} = \frac{5-3}{1-0} = 2$$

Figure 1A.1: Graph of the Linear Function Y = 3 + 2X



$$\text{Slope} = \frac{\Delta Y}{\Delta X} = \frac{5-3}{1-0} = 2$$

Slopes

- The slope is the same along a straight line.
- For the general form of the linear equation the slope equals b
- The slope can be positive (as in Figure 1A.1), negative (as in Figure 1A.2) or zero
- If the slope is zero, the straight line is horizontal with Y = intercept

Slope and Units of Measurement

- The slope of a function depends on the units in which X and Y are measured
- If the independent variable in the equation Y = 3 + 2X is income and is measured in hundreds of dollars, a $100 increase would result in 2 more units of Y

Slope and Units of Measurement

- If the same relationship was modeled but with X measured in single dollars, the equation would be Y = 3 + .02 X and the slope would equal .02

Changes in Slope

- In economics we are often interested in changes in the parameters (a and b of the general linear equation)
- In Figure 1A.2 the (negative) slope is doubled while the intercept is held constant
- In general, a change in the slope of a function will cause rotation of the function without changing the intercept

FIGURE 1A.2: Changes in the Slope of a Linear Function

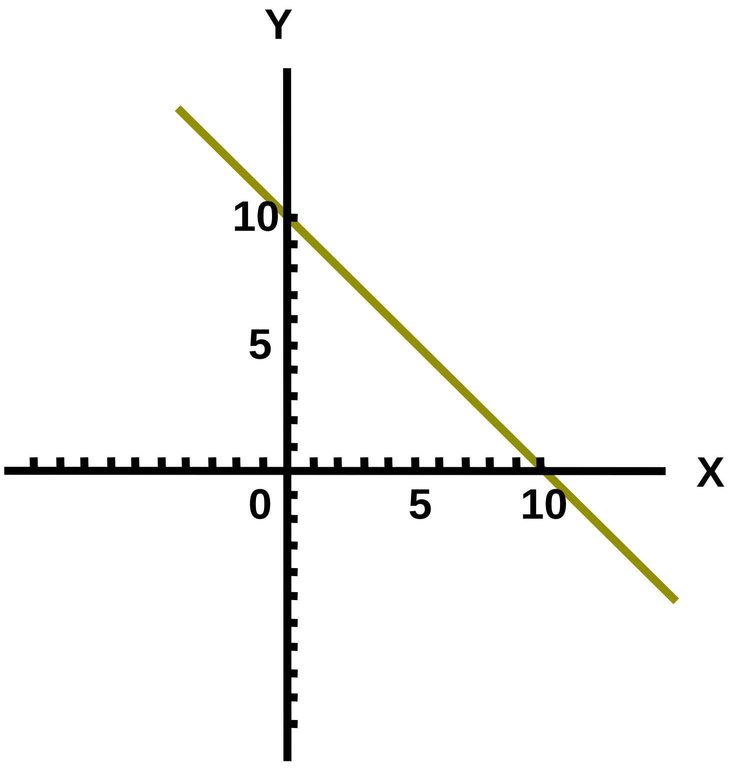
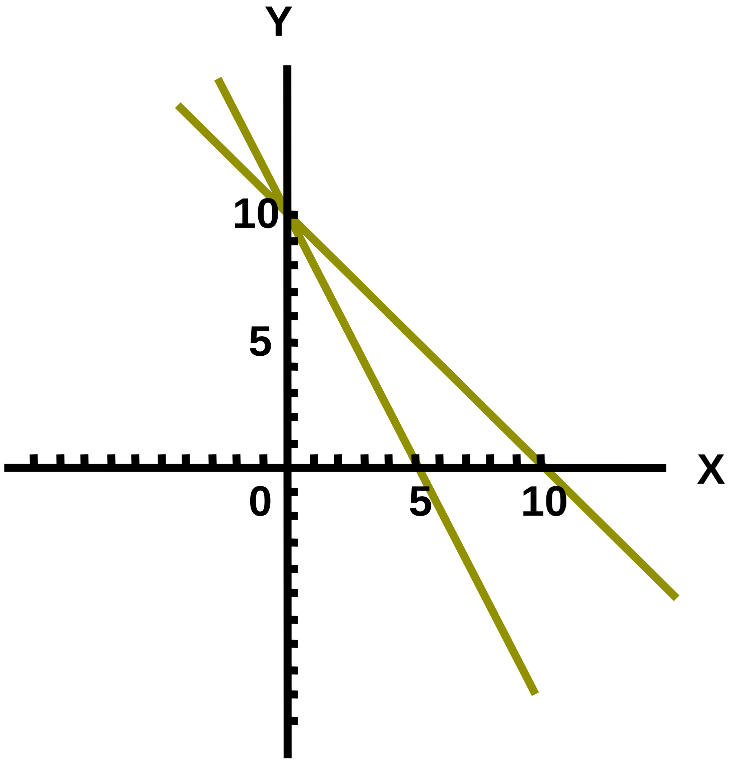


FIGURE 1A.2: Changes in the Slope of a Linear Function



Changes in Intercept

- When the slope is held constant but the intercept is changed in a linear function, this results in parallel shifts in the function
- In Figure 1A.3, the slope of all three functions is -1, but the intercept equals 5 for the line closest to the origin, increases to 10 for the second line and 12 for the third
  - These represent "Shifts" in a linear function.

FIGURE 1A.3: Changes in the Y-Intercept of a Linear Function

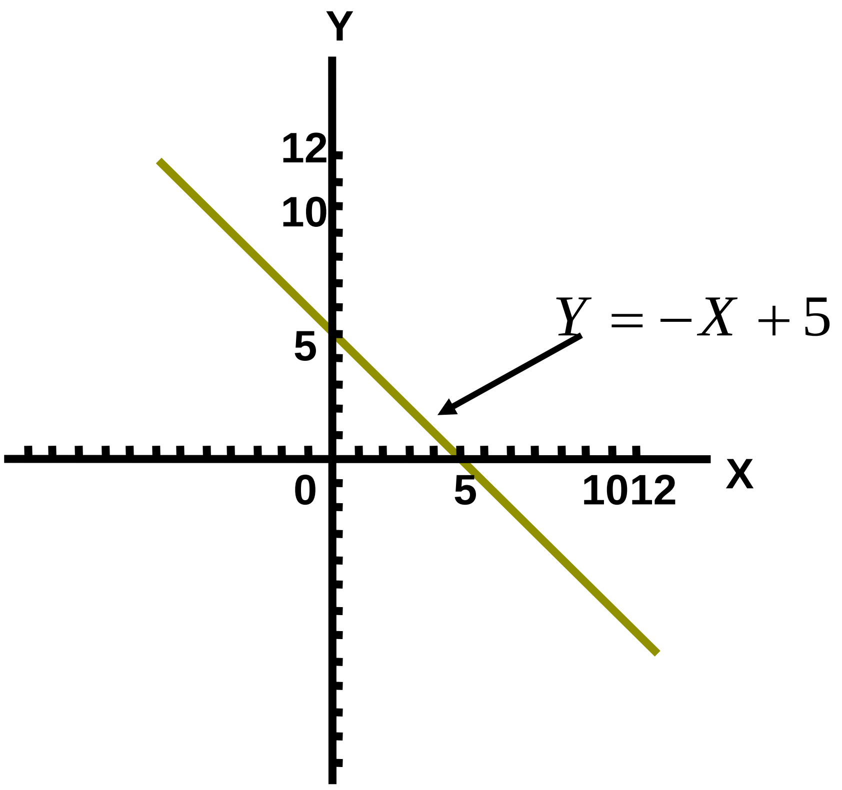


FIGURE 1A.3: Changes in the Y-Intercept of a Linear Function

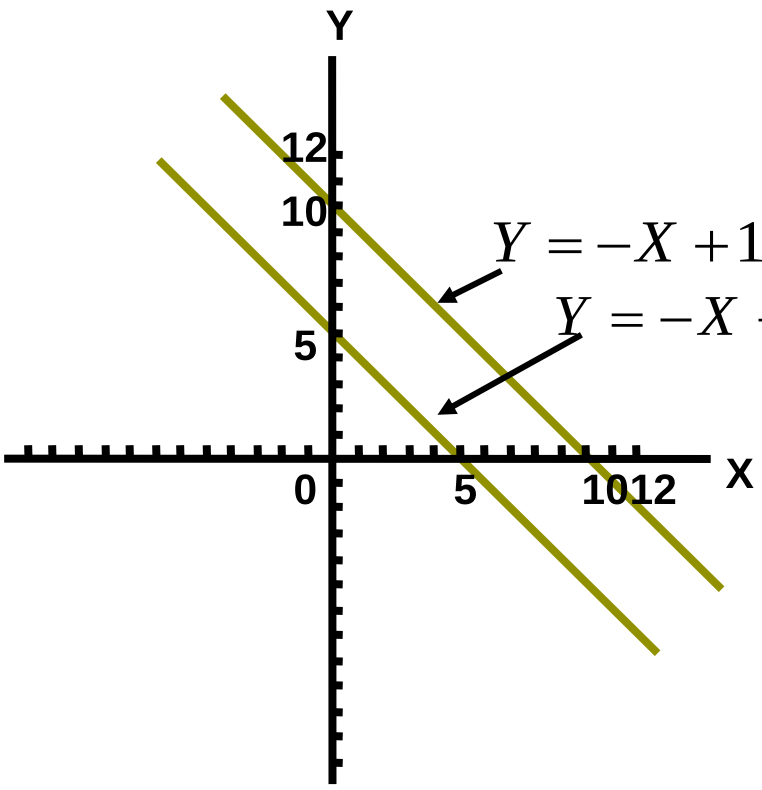
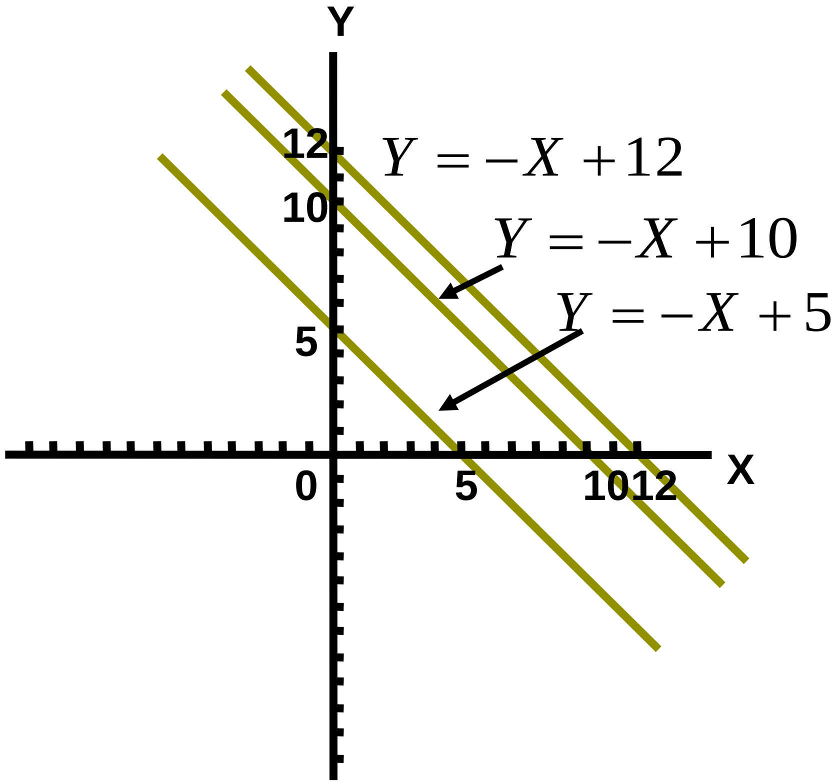


FIGURE 1A.3: Changes in the Y-Intercept of a Linear Function



Nonlinear Functions

- Figure 1A.4 shows the graph of the nonlinear function $Y = -X^2 + 15X$
- As the graph shows, the slope of the line is not constant but, in this case, diminishes as X increases
- This results in a concave graph which could reflect the principle of diminishing returns

FIGURE 1.A.4: Graph of the Quadratic Function $Y = X^2 + 15X$

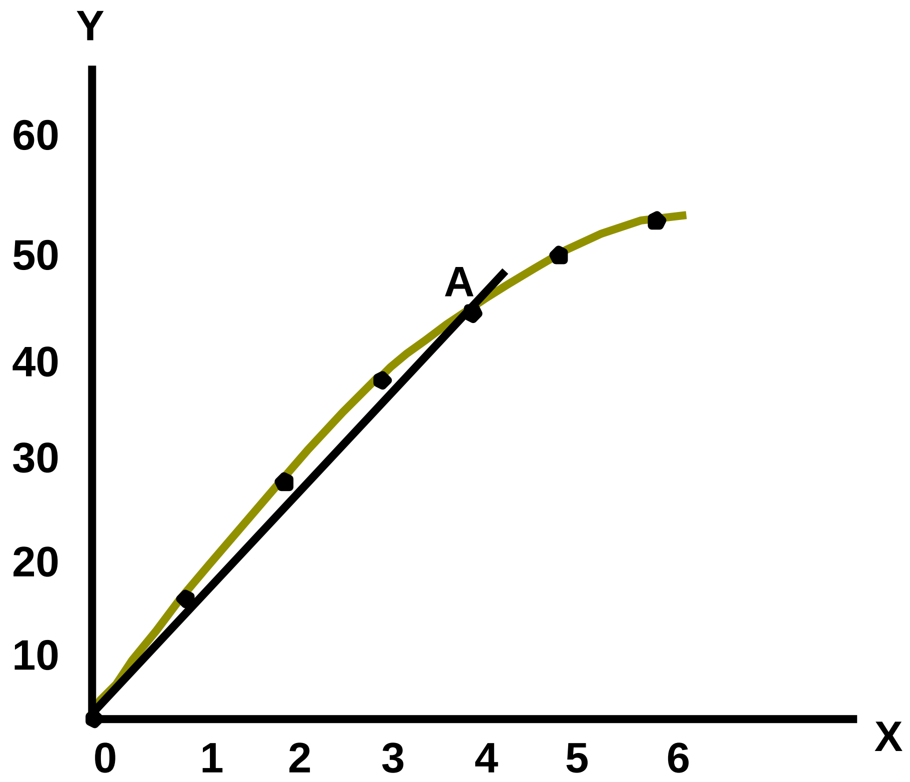
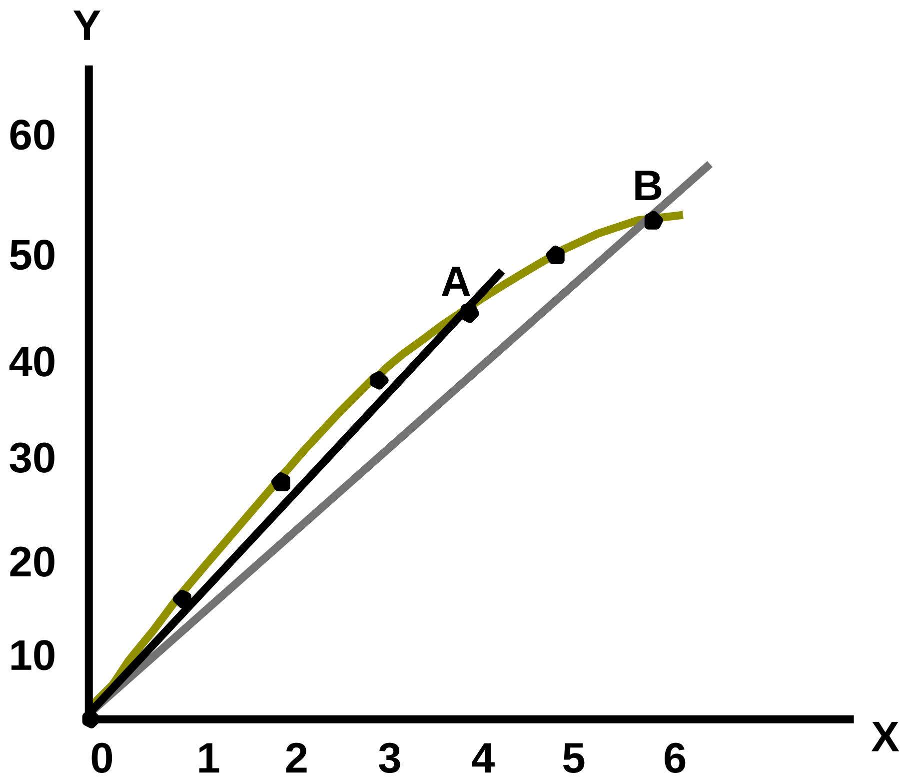


FIGURE 1.A.4: Graph of the Quadratic Function $Y = X^2 + 15X$



The Slope of a Nonlinear Function

- The graph of a nonlinear function is not a straight line
- Therefore it does not have the same slope at every point
- The slope of a nonlinear function at a particular point is defined as the slope of the straight line that is tangent to the function at that point.

Marginal Effects

- The **marginal effect** is the change in Y brought about by one unit change in X at a particular value of X (Also the slope of the function)
- For a linear function this will be constant, but for a nonlinear function it will vary from point to point

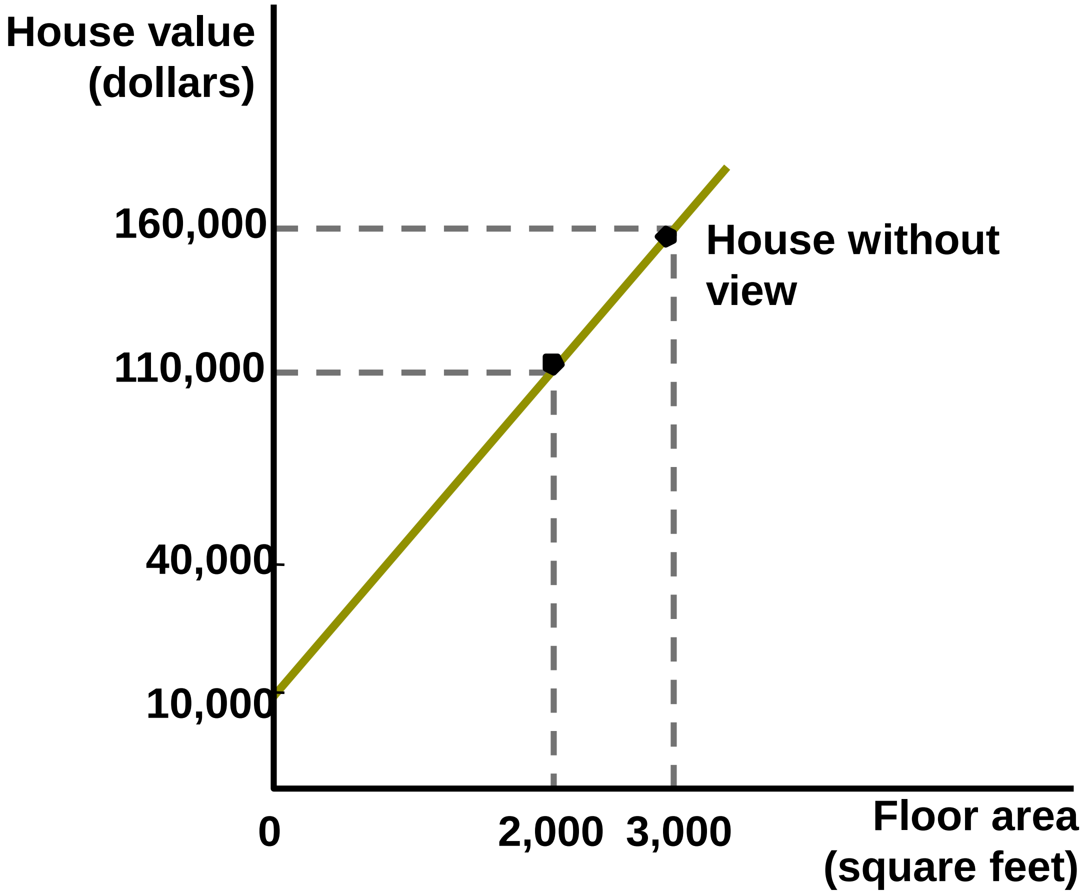
Average Effects

- The **average effect** is the ratio of Y to X at a particular value of X (the slope of a ray to a point)
- In Figure 1A.4, the ray that goes through A lies about the ray that goes through B indicating a higher average value at A than at B

APPLICATION 1A.1: Property Tax Assessment

- The bottom line in Figure 1 represents the linear function Y = \$10,000 + \$50X, where Y is the sales price of a house and X is its square footage
- If, other things equal, the same house but with a view is worth \$30,000 more, the top line Y = \$40,000 + \$50X represents this relationship

FIGURE 1: Relationship between the Floor Area of a House and Its Market Value



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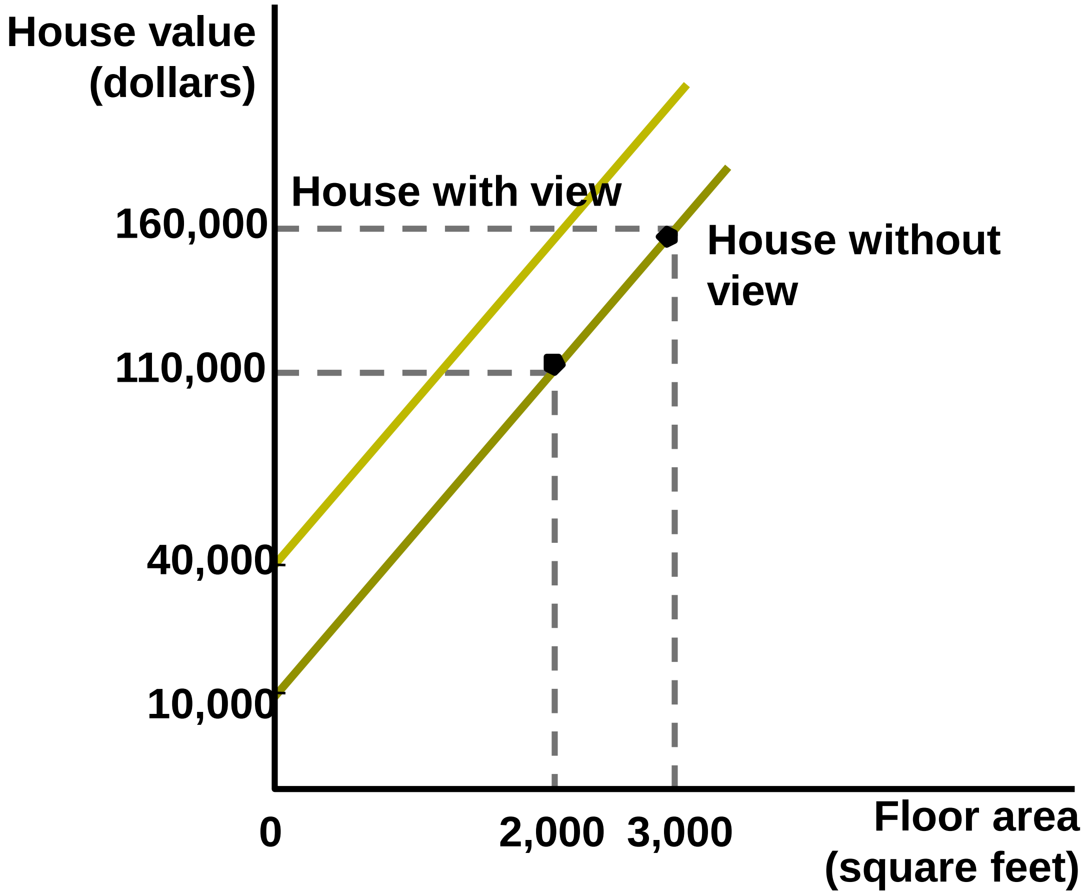


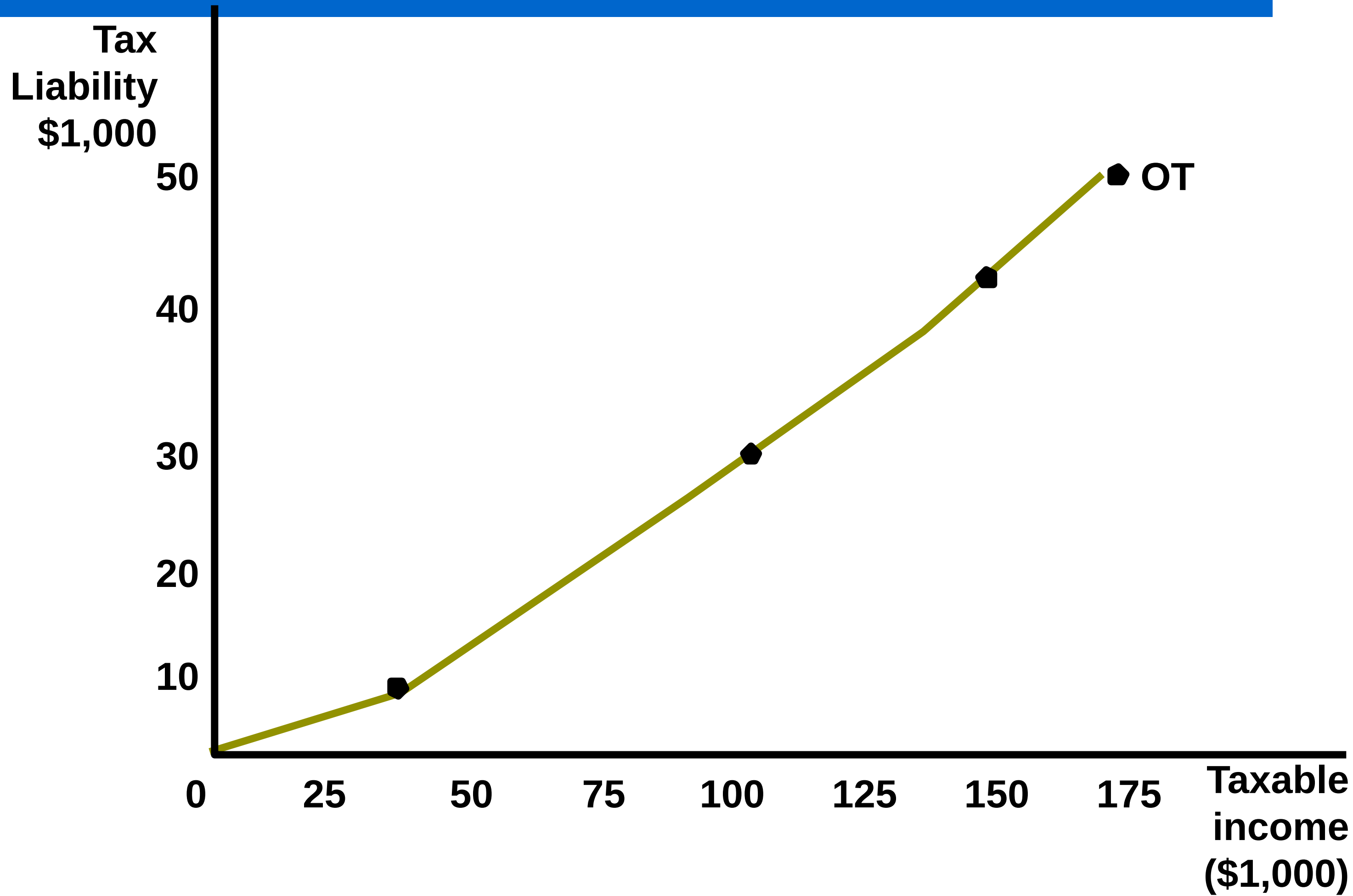
Calculus and Marginalism

- In graphical terms, the derivative of a function and its slope are the same concept
- Both provide a measure of the marginal inpact of X on Y
- Derivatives provide a convenient way of studying marginal effects.

APPLICATION 1A.2: Progressive and Flat Taxes

- Advocates of tax fairness argue that income taxes should progressive so that richer people should pay a higher fraction of their incomes in taxes
  - This is illustrated in Figure 1 by the nonlinear line OT that becomes steeper as taxable income increases
  - This represents an increasing marginal tax rate

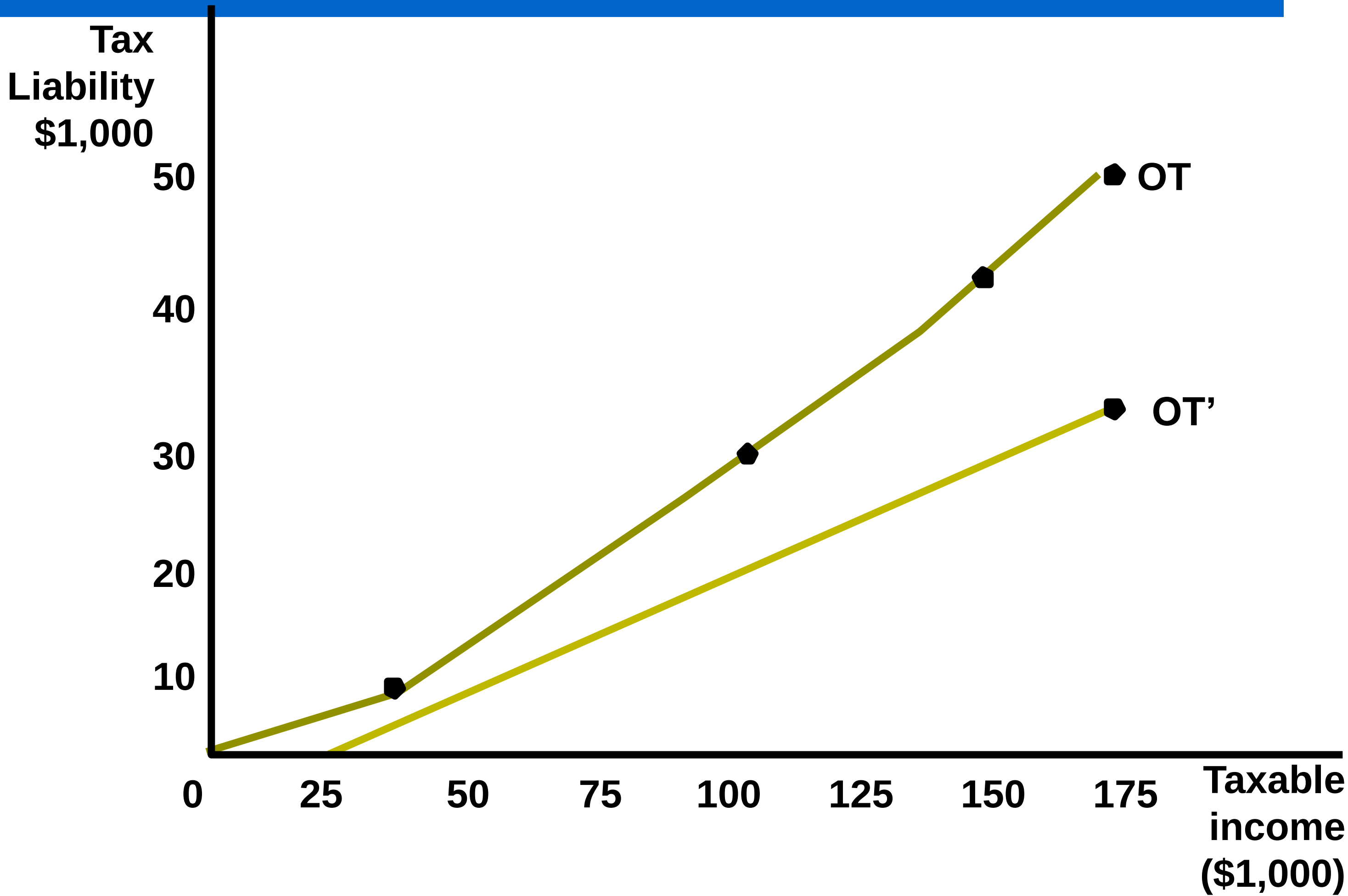
FIGURE 1: Progressive Rates Compared to a Flat Tax Schedule



APPLICATION 1A.2: Progressive and Flat Taxes

- Opponents of progressive taxes have argued for a flat tax
- The straight line OT' represents a proposal where the first $18,000 of taxable income would not be taxed with a flat tax of 17 percent on additional taxable income
  - This would also be progressive but not as much as in the current system

FIGURE 1: Progressive Rates Compared to a Flat Tax Schedule



Functions of Two or More Variables

- The dependent variable can be a function of more than one independent variable
- The general equation for the case where the dependent variable Y is a function of two independent variables X and Z is

$$Y = f(X, Z)$$

A Simple Example

- Suppose the relationship between the dependent variable (Y) and the two independent variables (X and Z) is given by

$$Y = X \cdot Z$$

- Some values for this function are shown in Table 1A.2

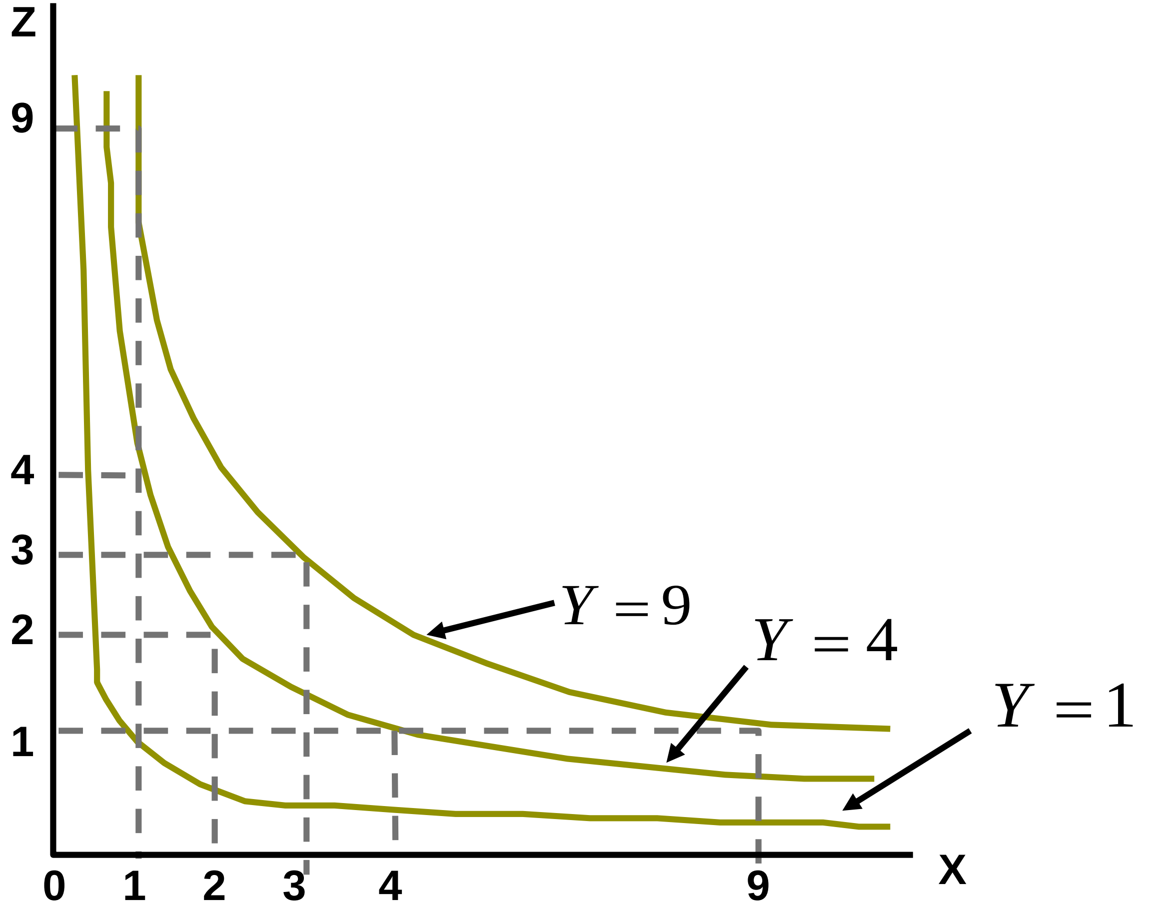
TABLE 1A.2: Values of X, Z, and Y that satisfy the Relationship Y = X·Z

X	Z	Y
1	1	1
1	2	2
1	3	3
1	4	4
2	1	2
2	2	4
2	3	6
2	4	8
3	1	3
3	2	6
3	3	9
3	4	12
4	1	4
4	2	8
4	3	12
4	4	16

Graphing Functions of Two Variables

- Contour lines are frequently used to graph functions with two independent variables
- Contour lines are lines in two dimensions that show the sets of values of the independent variables that yield the same value for the dependent variable
- Contour lines for the equation $Y = X \cdot Z$ are shown in Figure 1A.5

FIGURE 1A.5: Contour Lines for Y = X·Z



Simultaneous Equations

- These are a set of equations with more than one variable that must be solved together for a particular solution
- When two variables, say X and Y, are related by two different equations, it is sometime possible to solve these equations to get a set of values for X and Y that satisfy both equations

Simultaneous Equations

The equations [1A.17]

$$X + Y = 3$$

$$X - Y = 1$$

can be solved for the unique solution

$$X = 2$$

$$Y = 1$$

Changing Solutions for Simultaneous Equations

The equations [1A.19]

$$X + Y = 5$$

$$X - Y = 1$$

can be solved for the unique solution

$$X = 3$$

$$Y = 2$$

Graphing Simultaneous Equations

- The two simultaneous equations systems, 1A.17 and 1A.19 are graphed in Figure 1A.6
- The intersection of the graphs of the equations show the solutions to the equations systems
- These graphs are very similar to supply and demand graphs

Figure 1A.6: Solving Simultaneous Equations

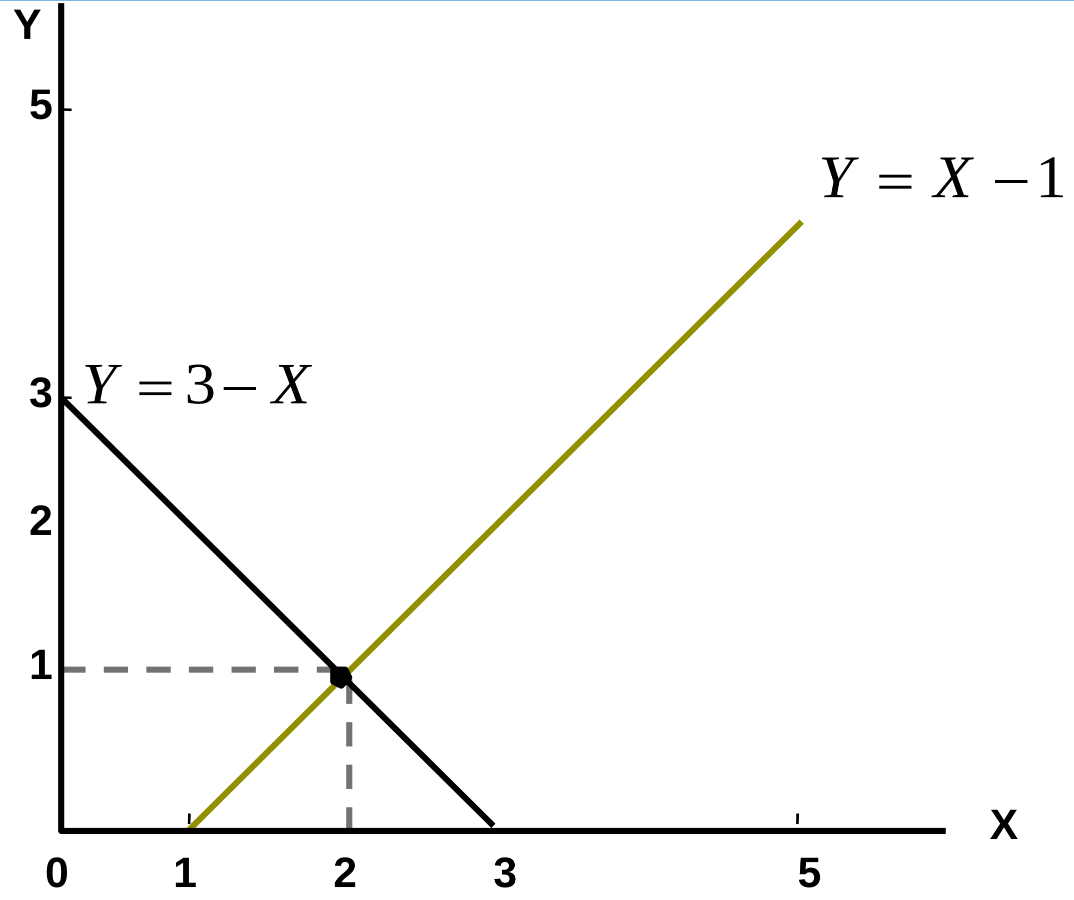
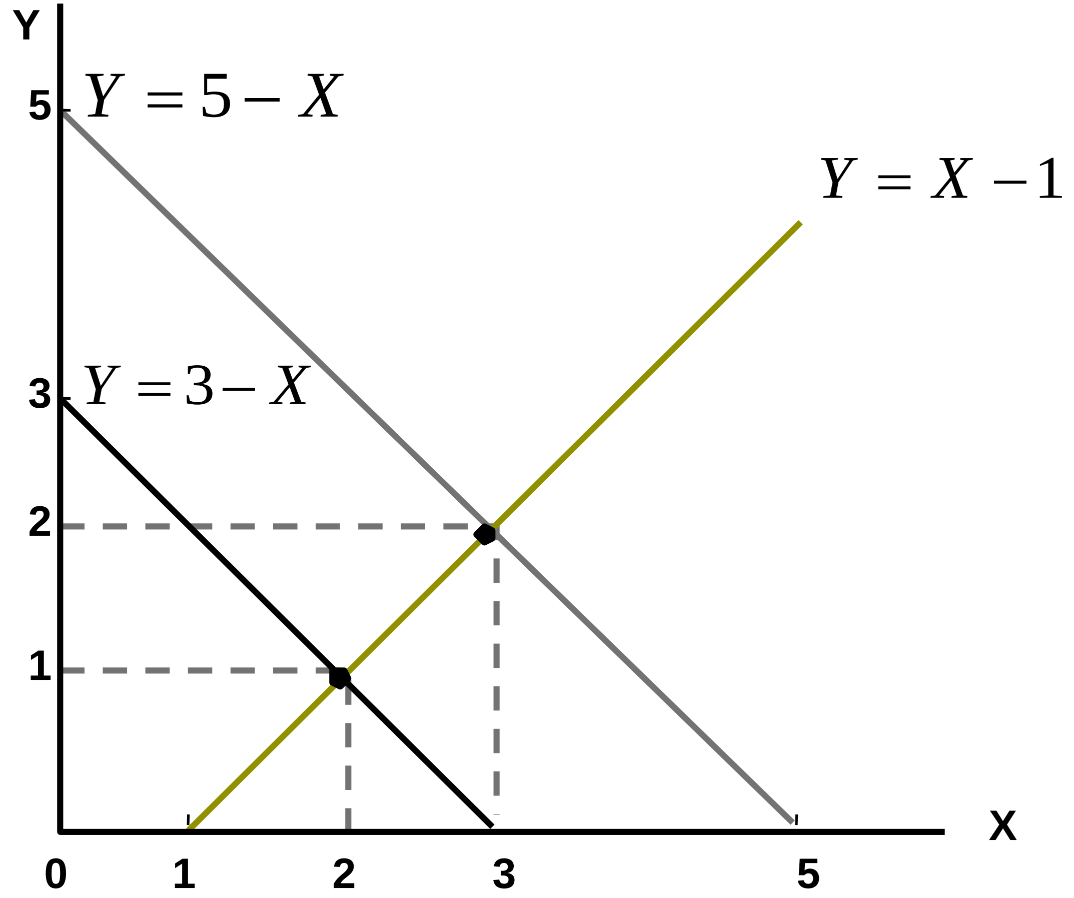


Figure 1A.6: Solving Simultaneous Equations



APPLICATION 1A.3: Can Iraq Affect Oil Prices?

Assume the demand for crude oil is given by

$$Q_D = 80 - 0.4P$$

where Q_D is crude oil demanded (in millions of barrels per day) and P price in dollars per barrel.

Assume the supply of crude oil is given by

$$Q_S = 55 + 0.6P$$

The solution to these equations, market equilibrium, is P = 25 and $Q_S = Q_D = 70$ and can be found by

$$80 - 0.4P = 55 + 0.6 \text{ or } \mathrm{P} = 25, \mathrm{Q} = 70$$

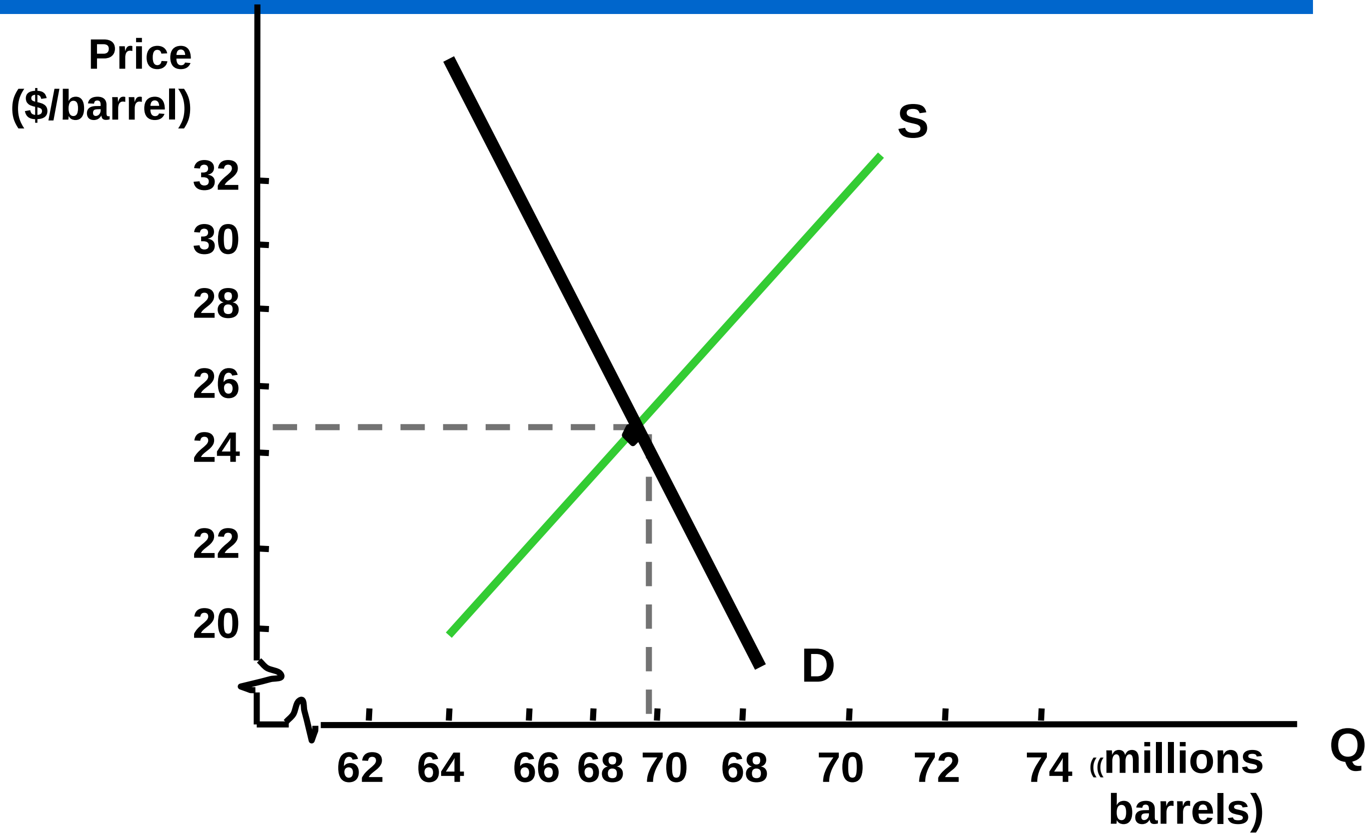
APPLICATION 1A.3:Can Iraq Affect Oil Prices?

Iraq produces about 2.5 million barrels of oil per day. The impact of the decision to sell no oil can be evaluated by assuming that the supply curve in Figure 1 shifts to S' whose equation is given by

$$Q_S = (55 - 2.5) + 0.6P = 52.5 + 0.6P$$

Repeating the algebra yields a new equilibrium, as shown in Figure 1, of P=27.50 and Q = 69. The reduction in oil supply raised the price and decreased consumption. The higher price caused non-OPEC producers to supply about 0.5 million additional barrels.

FIGURE 1: Effect of OPEC Output Restrictions on World Oil Market



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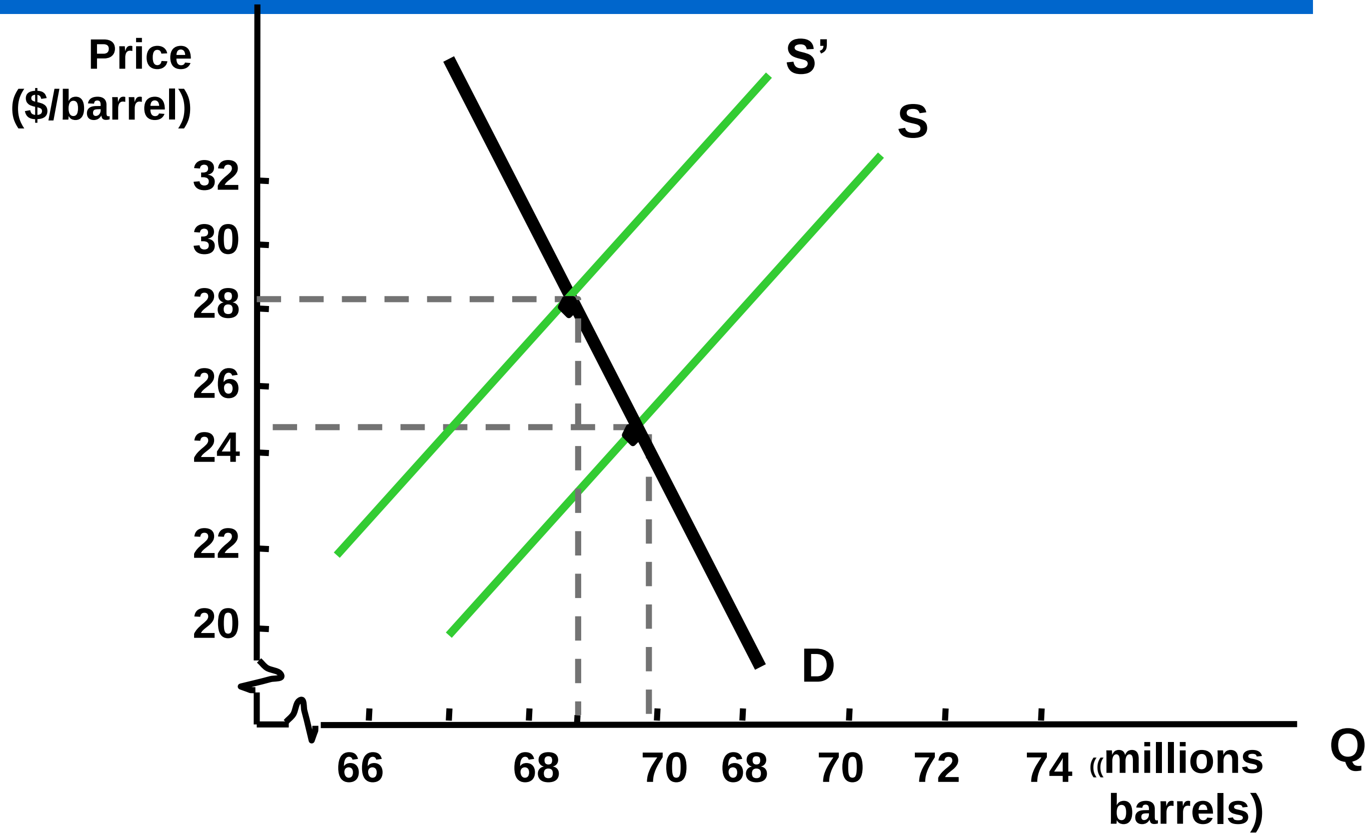


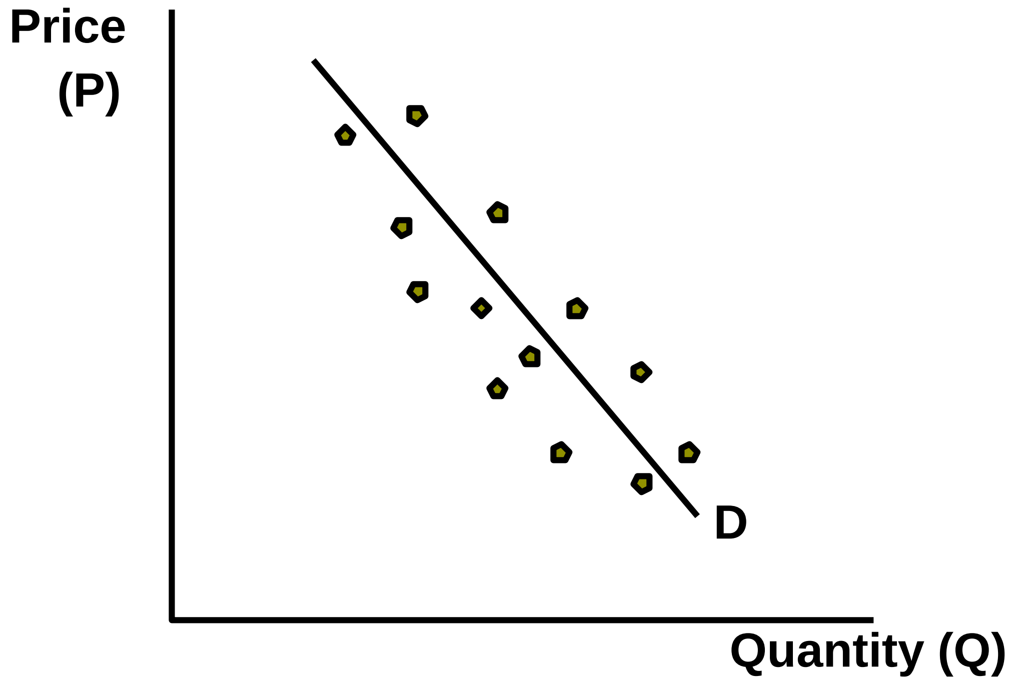
Empirical Microeconomics and Econometrics

- Economists test the validity of their models by looking at data from the real world
- Econometrics is used for this purpose
- Two important aspects of econometrics are
  - random influences
  - the *ceteris paribus* assumption

Random Influences

- No economic model exhibits perfect accuracy so actual price and quantity values will be scattered around the “true” demand curve
- Figure 1A.7 shows the unknown true demand curve and the actual points observed in the data from the real world
- The problem is to infer the true demand curve

FIGURE 1A.7: Inferring the Demand Curve from Real-World Data



Random Influences

- Technically, the problem is **statistical inference**: the use of actual data and statistical techniques to determine quantitative economic relationships
- Since no single straight line will fit all of the data points, the researcher must give careful consideration to the random influences to get the best line possible

The *Ceteris Paribus* Assumption

- To control for the "other things equal" assumption two things must be done
  - Data should be collected on all of the other factors that affect demand, and
  - appropriate procedures must be used to control for these measurable factors in the analysis
- Generally the researcher has to make some compromises which leads to many controversies in testing economic models